Fundamentals of Data Representation (Part 2)

ITS30505 Introduction to Computing

Lecture 8

Learning Outcomes

At the end of this lecture, you should be able to:

- represent signed decimal integers in binary using the two's complement system
- perform binary addition and subtraction between two binary numbers
- perform hexadecimal addition and subtraction between two binary numbers

Signed Binary Numbers

Up until now, we have assumed all binary numbers have positive values. There are a number of methods to represent both positive and negative numbers. We will consider:

- one's complement
- two's complement

In addition, we will assume that these operations will be carried out in an 8-bit register (i.e., decimal numbers between -128 and 127).

One's Complement System

In one's complement, each digit in the binary number is inverted (i.e., 0 becomes 1, and 1 becomes 0).

90	0 1 0 1 1 0 1 0
−90	1 0 1 0 0 1 0 1

However, one major issue lies with the number 0. Using the same concept, you can easily create -0.

0	0 0 0 0 0 0 0 0
−0	1 1 1 1 1 1 1 1

In the real number system, there is no such thing as -0!

Two's Complement System

Two's complement solves the -0 problem by adding 1 after inverting all the bits.

Original number (0)	0 0 0 0 0 0 0 0 ↓
1's complement system	1 1 1 1 1 1 1 1
2's complement system (add 1)	0 0 0 0 0 0 0 0

0 will now retain the same binary value after using the two's complement system.

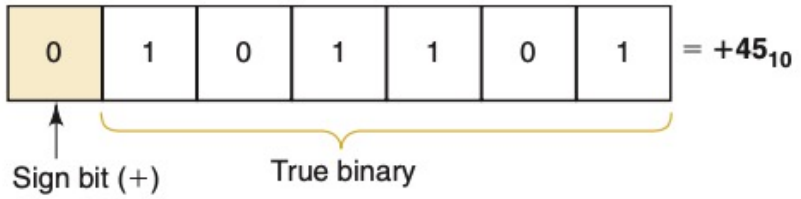
Using the previous example to get -90 in the two's complement system:

Original binary number (90)	0 1 0 1 1 0 1 0 ↓
1's complement of binary number (-90)	1 0 1 0 0 1 0 1
2's complement system of binary number (add 1)	1 0 1 0 0 1 1 0

Representing Signed Numbers using Two's Complement

The two's complement system works like this:

- If the number is positive, the magnitude is represented in its true binary form, and a sign of bit 0 is placed in front of the MSB (most significant bit/the leftmost bit).



- If the number is negative, the magnitude is represented in its 2's complement form, and a sign bit of 1 is placed in front of the MSB.

Representing Signed Numbers using Two's Complement

- The two's complement system can be used on a negative number to obtain its positive number variant.
- One example of a quick way to obtain the negative decimal number from a negative binary number is by doing the following:

To check if $1010\ 0110_2$ is indeed -90_{10},

$$1010\ 0110_2 = -128 + 32 + 4 + 2$$
$$= -90_{10}$$

The sign bit at the end is represented with a negative counterpart on the first bit (in this case, 2^7 or 128).

–128	64	32	16	8	4	2	1
1	0	1	0	0	1	1	0

Original binary number (90)	0 1 0 1 1 0 1 0
1's complement of binary number (-90)	↓ 1 0 1 0 0 1 0 1
2's complement system of binary number (add 1)	1 0 1 0 0 1 1 0

Binary Addition

Case 1: Two Positive Numbers

Example: Add $+37$ and $+58$ in binary.

Binary Addition

Case 2: Two Large Positive Numbers

Example: Add $+82$ and $+69$ in binary.

Binary Addition

Case 3: Positive Number and Smaller Negative Number

Example: Add $+19$ and -4 in binary.

Binary Addition

Case 4: Positive Number and Larger Negative Number

Example: Add $+9$ and -14 in binary.

Binary Addition

Case 5: Two Negative Numbers

Example: Add -9 and -14 in binary.

Binary Addition

Case 6: Equal and Opposite Numbers

Example: Add $+14$ and -14 in binary.

Binary Subtraction using Two's Complement

Example: Carry out the subtraction $95 - 68$ in binary.

Binary Subtraction using Two's Complement

Example: Carry out the subtraction $49 - 80$ in binary.

Hexadecimal Addition

- Add the hex digits in decimal.
- If the sum if 15 or less, express directly in hex digits.
- If the sum is greater than 15, subtract 16 and carry 1 to the next position.

Example: $3\mathrm{E}91_{16} + 2\mathrm{F}93_{16}$

Caution: Be extra mindful to not carry over at 10, it must carry over at 16!

Hexadecimal Subtraction

Two methods:

- Deduct the numbers regularly like with decimal numbers, but keep in mind to carry over 16, when necessary, instead of 10
- Use the two's complement on the subtracting number

Hexadecimal Subtraction (Regular Method)

Example: $91\mathrm{B}_{16} - 6F2_{16}$

Caution: Be extra mindful to not carry over 10, it must carry over 16!

Hexadecimal Subtraction using 2's Complement

Example: $91\mathrm{B}_{16} - 6F2_{16}$

To get 2's complement of $6F2_{16}$:

Summary

This lecture has covered the following:

- Signed Binary Numbers: One's Complement & Two's Complement Systems
- Binary Arithmetic Operations: Binary Addition & Binary Subtraction
- Hexadecimal Arithmetic Operations: Hexadecimal Addition & Hexadecimal Subtraction

Source: http://www.devtopics.com/best-programming-jokes/

References

- Widmer, N. S., Moss, G. L. & Tocci, R. J. (2017). *Digital Systems: Principles and Applications* (12th Edition). Pearson Education Limited.
- Watson, D. & Williams, H. (2019). *Cambridge International AS & A Level Computer Science.* Hodder Education.