Karnaugh Map

ITS30505 Introduction to Computing

Lecture 11

Learning Outcomes

At the end of this lecture, you should be able to:

- utilize the Karnaugh map method to simplify Boolean expressions of 2, 3 and 4 variables

Karnaugh Maps

In previous activities, it was frequently necessary to simplify Boolean expressions. Sometimes, this can be a long and complex process. **Karnaugh maps** (often shortened as **K-maps**) were developed as another method of simplifying logic expressions/circuits.

- Karnaugh maps represent the contents of a truth table as a grid or 'map'.

Example 1: Produce a Boolean expression for the truth table for the NAND gate.

Input		**Output**
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Example 1

Example: Produce a Boolean expression for the truth table for the NAND gate.

Input		Output
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Karnaugh Map Rules

As you might expect, there are a few rules governing Karnaugh maps.

- The values along the top and bottom follow Gray code rules.
- Only cells containing 1 are taken account of.
- Groups can be a row, a column, or a rectangle.
- Number of 1s should be in powers of 2 (i.e., $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, and so on).
- Groups should be as large as possible.
- Groups may overlap within the above rules.
- Single values can be regarded as a group even if they cannot be combined with other values to form a larger group.
- The final Boolean expression can only consider those values which remain *constant* within the group (that is, remain a 1 or a 0 throughout the group).

Example 2

Produce a Boolean expression for the following truth table.

Input			Output
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Example 2: Solution

Example 3

Produce a Boolean expression for the following truth table.

Input				Output
A	B	C	D	X
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Example 3: Solution

Possible K-map Options

This gives us the value D since the values of A and B change and the value of C changes (0 to 1); only D is constant at 1.

CD \ AB	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1	0	0	0
$\bar{C}D$	1	1	1	1
CD	1	1	1	1
$C\bar{D}$	0	0	0	0

Possible K-map Options

Columns 1 and 4 can be joined to form a *vertical cylinder*. The values of both C and D change, the value of A changes, the value of B is constant at 0 giving: $\bar{B}$

CD \ AB	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1	0	0	1
$\bar{C}D$	1	0	0	1
CD	1	0	0	1
$C\bar{D}$	1	0	0	1

Possible K-map Options

The two 1-values can be combined to form a *horizontal cylinder*; values of A and B are constant at 0 and 1 respectively; the value of D is constant at 0; value of C changes from 0 to 1; giving: $\bar{A} \cdot B \cdot \bar{D}$

CD \ AB	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	1	0	0
$\bar{C}D$	0	0	0	0
CD	0	0	0	0
$C\bar{D}$	0	1	0	0

Possible K-map Options

The four 1-values can be combined at the four corners; value B is constant at 0 and value D is also constant at 0, giving: $\bar{B} \cdot \bar{D}$

CD \ AB	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1	0	0	1
$\bar{C}D$	0	0	0	0
CD	0	0	0	0
$C\bar{D}$	1	0	0	1

Summary

This lecture has covered the following:

- Karnaugh Maps
- Rules Governing Karnaugh Maps
- Simplifying Boolean Expressions using Karnaugh Maps

&&	!alive	alive
!dead		
dead		

Source: https://www.reddit.com/r/ProgrammerHumor/comments/68yyn4/boolean_logic/

References

- Watson, D. & Williams, H. (2019). *Cambridge International AS & A Level Computer Science*. Hodder Education.
- Widmer, N. S., Moss, G. L. & Tocci, R. J. (2017). *Digital Systems: Principles and Applications* (12th Edition). Pearson Education Limited.